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The Mixing Matrix for a 3+2 model

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Abstract

Recently, the MiniBooNE (MB) experiment denied the LSND results and, what is more unexpected, results were obtained at low energies. But the MiniBooNE results are in perfect agreement with models with more than one sterile neutrino. Its should be noted, also, that scenarios with two sterile neutrinos offer rich and interesting phenomenology, for example, the possibility to explain the asymmetry matter anti-matter in the universe through leptogenesis. Thus the mass matrix obteined in the 3+2 model is still considered, therefore, this work is dedicated to the study, even though initial one, of mixing matrix obtaned from this mass matrix. The mass matrix considerate in this work is obtained by a specific model based in extention of the standard model with addition of right-hand neutrinos and gauge symmetries. The mixing matrix obtained have phases of CP violation and is characterized in terms of sinos and cossinos.

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Nowadays there is convincing evidence for solar and atmospheric neutrino oscillations. A summary of the data, taken from Ref.[1] is shown bellow.

$$
\begin{aligned}
&7.7 \times 10^{-5} < (\Delta m^2_{sun})_{\mathrm{LA}} < 8.4 \times 10^{-5}\ \mathrm{eV}^2;\\
&1.9 \times 10^{-3} < \Delta m^2_{atm} < 3.0 \times 10^{-3}\ \mathrm{eV}^2;\\
&0.82 < \sin^2 2\theta_{12} < 0.89;\\
&\sin^2 2\theta_{23} > 0.92;\\
&\sin^2 2\theta_{13} < 0.19.
\end{aligned}
$$

Recently, the first results from the MiniBooNE (MB) experiment [2] at Fermilab have been released on a search for the $\nu_\mu \to \nu_e$ oscilation appearance with a baseline of 540 m and a mean neutrino energy of about 700 MeV. The primary purpose of this experiment is to test the evidence of $\overline{\nu_\mu} \to \overline{\nu_e}$ transitions, reported by LSND experiment at Los Alamos [3] with a very similar L/E range. However, such results were denied by the MiniBooNE and, what is more unexpected, they were obtained at low energies. This results are a challenge for neutrino phenomenology, since the mass-squared differences required to explain the solar, atmospheric, LSND and MiniBooNe experimental datas in terms of neutrino oscillations, differ of one another by various orders of magnitude. Which means, there is no way to explain all these three signals invoking only oscillations among the usal three neutrinos in the standard model. Based on that, one had to invoke an extension of the standard model. As a result we have to change the three-neutrino mixing scenario, introducing sterile neutrinos to the content of matter of the standard model.

In the light of the new experimental data, Maltoni [4] revisited the global neutrino oscillation fitting to short-baseline neutrino data, and concluded that models like (3+1) [5] become even more disfavored with the new data. However, MiniBooNE datas are in perfect agreement with models with more than one sterile neutrino [6] disregarding experimental data of appearance and disappearance, which can be explain thanks to the possibility of CP violation available in such oscillation schemes, among other possibilities. Its should be noted that scenarios with two sterile neutrinos offer rich and interesting phenomenology, for example, the possibility to explain the asymmetry matter anti-matter in the universe through leptogenesis. [7]

As we can see in recent articles like [8], the mass matrix obtained in the 3+2 model is still considered. In this way, this article is dedicated to the study, even though initial one, of mixing matrix obtained from such mass matrix. The outline of this paper is as follows. First we present a model that obtain, in a natural way the mass matrix of the 3+2 type, then we dedicate ourselves to studying the mass matrix obtained, getting their eigenvalues and eigenvectors; finally we present the mixing matrix written in terms of sines and cossines.

In an attempt to explain why the number of right-handed neutrino is completely arbitrary, in the context of the electroweak standard model (SM), the authors in the ref.[9], propose a new model considering the fact that sterile neutrinos are sterile with respect to the interaction of the SM. Such neutrinos do not contribute to the anomaly cancellation, and hence nothing determine their number, but if there are interactions beyond the usual ones, right-handed neutrinos may be no more sterile with respect to these new interactions and their number have to be constrained by the anomaly cancellations, concerning all the new interactions. A natural possibility is that global automatic symmetries of the standard model are in fact gauge symmetries when the model is embedded into a larger gauge symmetry.

Summing up this work, the authors show that gauging the $B-L$ symmetry and its anomaly cancellations plus the requirement that the total lepton number be integer, imply that only three sterile neutrinos can be added to the minimal representation content of the SM. Two situations arise: i) three identical sterile neutrinos i.e., all of them having total lepton number $L=1$; ii) two sterile neutrinos with $L=4$ and the third one with $L=-5$. In the latter case it is possible, depending on the scalar sector, to mimic 3+1 and 3+2 neutrino mixing schemes, depending on the scalar doublets or on which scalar singlet triggers the spontaneous breakdown of the $B-L$ symmetry. Therefore, it is precisely in the latter scheme that we are interested in this work, with this model we are capable of give a greater motivation for a detailed study of the matrix mass of the type 3+2.

The model has the same matter content of the standard model plus the three right hand neutrinos ($N_R = 3$ considering the second solution), two scalars for generating the Majorana masses of the right-handed neutrinos. Thus the scalar doublets have $Y=-1$: Φ_1 with $Y'=-4$, and $(B-L)=+3$, and Φ_2 with $Y'=5$, and $(B-L)=-6$; and two scalar singlets $(Y=0)$: ϕ_1 with $Y'=-(B-L)=-8$, ϕ_2 with $Y'=-(B-L)=10$, and ϕ_3 with $Y'=-(B-L)=1$. Thus the yukawa interactions for this model is given by:

$$
\begin{aligned}
-\mathcal{L}^{\nu}_{\text{yukawa}} &= \overline{\Psi}_{aL} G^{D}_{am} \Phi_1 n_{mR} + \overline{\Psi}_{aL} G^{D}_{a3} \Phi_2 n_{3R} + \phi_1 \, \overline{(n_{mR})^c} \, G^{M}_{mn} \, n_{nR} \\
&+ \ \phi_2 \, \overline{(n_{3R})^c} \, G^{M}_{33} n_{3R} + \phi_3 \overline{(n_{mR})^c} \, G^{M}_{m3} \, n_{3R} + H.c., \qquad (1)
\end{aligned}
$$

in which $m, n = 1, 2$.

If all the interactions in Eq. (1) are in fact allowed, we have a general 3+3 scheme. However, if we do not introduce the singlet ϕ_3, we have the following situations: if Φ_1 is not introduced, the resulting scheme is 3+1, while if Φ_2 is not present, the scheme becomes 3+2. These schemes also arise, if $\langle\phi_2\rangle \gg \langle\phi_{1,3}\rangle \gtrsim \langle\Phi_{1,2}\rangle$, or $\langle\phi_1\rangle \gg \langle\phi_{2,3}\rangle \gtrsim \langle\Phi_{1,2}\rangle$, respectively. Notice also that not all scalar singlets need to get a nonzero VEV. For a more detailed analysis see ref. [9].

In this work we take the model above as a motivation. Note that the mass matrix obtained directly from eq.(1) is a 6×6, and because of the analyses of the potential, the terms that came from the interaction with the Φ_2 and ϕ_2 can be removed from the analyses. Thus the mass matrix that is considered now for the neutrinos is a mimic of the mass matrix of the 3+2 model. The mass matrix is given by:

$$
M^{\nu} = \begin{pmatrix}
0 & 0 & 0 & a_e & b_e \\
0 & 0 & 0 & a_\mu & b_\mu \\
0 & 0 & 0 & a_\tau & b_\tau \\
a_e & a_\mu & a_\tau & M_{11} & M_{12} \\
b_e & b_\mu & b_\tau & M_{21} & M_{22}
\end{pmatrix}, \qquad (2)
$$

in which $\frac{G^{D}_{a1} v_1}{\sqrt{2}} = a_\alpha$ and $\frac{G^{D}_{a2} v_1}{\sqrt{2}} = b_\alpha$ where $a = e, \mu, \tau$. In the ref. [8], the entries M_{12} and M_{21} are iqual to zero. For simplicity we first consider the model that does not take into account at all the majorana mass for the right hand neutrinos, to do this we just not introduce the singltes ϕ_i in the eq.(1). Thus we are capable of obtain the eigenvalues of this matrix, the λ's, and we can define the mass of the neutrinos as: $m_1 = \lambda_1 = 0$, $m_2 = \lambda_2$, $m_3 = \lambda_3 = -\lambda_2$, $m_4 = \lambda_4$, $m_5 = \lambda_5 = -\lambda_4$. Note that there is a problem of mass degeneration at the tree level, but this can be easily corrected if we consider the majorana mass terms in the matrix or by radioactive corrections, for example. The mixing matrix is built through the eigenvectors of the mass matrix, that is, the eigenvectors form the columns of the mixing matrix. Thus we have to obtain it.

To $\lambda_1 = 0$ the eigenvectors are given by:

$$X_1^1 = \frac{(\overrightarrow{a} \times \overrightarrow{b})_e}{\sqrt{|\overrightarrow{a} \times \overrightarrow{b}|}} \ , \quad X_2^1 = \frac{(\overrightarrow{a} \times \overrightarrow{b})_\mu}{\sqrt{|\overrightarrow{a} \times \overrightarrow{b}|}} \ , \quad X_3^1 = \frac{(\overrightarrow{a} \times \overrightarrow{b})_\tau}{\sqrt{|\overrightarrow{a} \times \overrightarrow{b}|}} \ , \quad X_4^1 = X_5^1 = 0 \qquad (3)$$

to $\lambda_i \neq 0$, we have the eigenvectors:

$$\begin{aligned}
X_1^i &= \frac{a_e A_i + b_e}{[(aA_i + b)^2 + (A_i^2 + 1)\lambda_i^2]^{\frac{1}{2}}} \ , \quad X_2^i = \frac{a_\mu A_i + b_\mu}{[(aA_i + b)^2 + (A_i^2 + 1)\lambda_i^2]^{\frac{1}{2}}} \\
X_3^i &= \frac{a_\tau A_i + b_\tau}{[(aA_i + b)^2 + (A_i^2 + 1)\lambda_i^2]^{\frac{1}{2}}} \ , \quad X_4^i = \frac{\lambda_i A_i}{[(aA_i + b)^2 + (A_i^2 + 1)\lambda_i^2]^{\frac{1}{2}}} \\
X_5^i &= \frac{\lambda_i}{[(aA_i + b)^2 + (A_i^2 + 1)\lambda_i^2]^{\frac{1}{2}}}. \qquad (4)
\end{aligned}$$

in which A_i é is given by:

$$A_i = \frac{\lambda_i^2 - b^2 - \overrightarrow{a}.\overrightarrow{b}}{a^2 + \overrightarrow{a}.\overrightarrow{b} - \lambda_i^2}, \qquad (5)$$

and we define $a^2 = a_e^2 + a_\mu^2 + a_\tau^2$; $b^2 = b_e^2 + b_\mu^2 + b_\tau^2$, $\overrightarrow{a}.\overrightarrow{b} = a_e b_e + a_\mu b_\mu + a_\tau b_\tau$ and $\overrightarrow{a} \times \overrightarrow{b} = (a_\mu b_\tau - a_\tau b_\mu)_e + (a_\tau b_e - a_e b_\tau)_\mu + (a_e b_\mu - a_\mu b_e)_\tau$.

Thus the mixing matrix has the following form:

$$R = \Theta \begin{pmatrix} X_1^1 & X_1^2 & X_1^2 & X_1^4 & X_1^4 \\ X_2^1 & X_2^2 & X_2^2 & X_2^4 & X_2^4 \\ X_3^1 & X_3^2 & X_3^2 & X_3^4 & X_3^4 \\ X_4^1 & X_4^2 & -X_4^2 & X_4^4 & -X_4^4 \\ X_5^1 & X_5^2 & -X_5^2 & X_5^4 & -X_5^4 \end{pmatrix}, \qquad (6)$$

in which $\Theta = diag(1, 1, i, 1, i)$ [10]. This matrix is one of the possible solutions to correct the signal of the neutrinos mass, but not the only one, we can modify the signal from the mass defining the neutrino field that corresponds to the negative mass as $\gamma_5 \nu_i$, as well. However, the basic idea in this report is to characterize the mixing matrix. Note that this mixing matrix is unitary by construction, but it is a very particular one. Thus, considering the properties of this matrix it is possible to write it in the present form:

$$R=\Theta\begin{pmatrix} c_\rho e^{-i\sigma} & \frac{-s_\rho c_\gamma}{\sqrt{2}} & \frac{-s_\rho c_\gamma}{\sqrt{2}} & \frac{-s_\rho s_\gamma}{\sqrt{2}} & \frac{-s_\rho s_\gamma}{\sqrt{2}} \\ s_\rho c_\beta & \frac{c_\rho c_\gamma c_\beta - s_\beta s_\gamma}{\sqrt{2}}e^{i\sigma} & \frac{c_\rho c_\gamma c_\beta - s_\beta s_\gamma}{\sqrt{2}}e^{i\sigma} & \frac{c_\rho s_\gamma c_\beta - s_\beta c_\gamma}{\sqrt{2}}e^{i\sigma} & \frac{c_\rho s_\gamma c_\beta - s_\beta c_\gamma}{\sqrt{2}}e^{i\sigma} \\ s_\rho s_\beta & \frac{c_\rho s_\gamma c_\beta - c_\beta s_\gamma}{\sqrt{2}}e^{i\sigma} & \frac{c_\rho s_\gamma c_\beta - c_\beta s_\gamma}{\sqrt{2}}e^{i\sigma} & \frac{c_\rho s_\gamma s_\beta - c_\beta c_\gamma}{\sqrt{2}}e^{i\sigma} & \frac{c_\rho s_\gamma s_\beta - c_\beta c_\gamma}{\sqrt{2}}e^{i\sigma} \\ 0 & \frac{s_\alpha}{\sqrt{2}} & \frac{-s_\alpha}{\sqrt{2}} & \frac{s_\eta}{\sqrt{2}} & \frac{-s_\eta}{\sqrt{2}} \\ 0 & \frac{c_\alpha}{\sqrt{2}} & \frac{-c_\alpha}{\sqrt{2}} & \frac{c_\eta}{\sqrt{2}} & \frac{-c_\eta}{\sqrt{2}} \end{pmatrix}. \tag{7}$$

In which we have to impose that:

$$cos(a+b)=cos(a)cos(b)-sen(a)sen(b)\ ,\ \alpha+\eta=\frac{\pi}{2}+k\pi \tag{8}$$

These conditions are the ones that keep the unitarity of this matrix. Also, note that we can associate $X_1^1=c_\rho$, $X_2^1=c_\beta s_\rho$, $X_2^1=s_\beta s_\rho$ like was made in the ref. [11]. But the others terms are very dificult to associate. We also have the phase of CP violation on this matrix. With this matrix we have a problem of degeneration of the neutrinos mass, because of this we are capable to explain only the differences of mass for the solar and atmosferic datas. Thus an analysis of the complete (3+2) mass matrix is necessary and is going to be presented elsewhere. However, note that the two differences in mass can explain the atmospheric and solar datas. This mixing matrix has CP violation phases, and we still have free parameters.

In summary, in this report we have obtained a characterized mixing matrix with CP violation phase in a 3+2 model which, so far, is the best model to explain all the current experimental data, since it is still allowed by the global neutrino data, including MiniBooNE. [7]. The disagreement between MiniBooNE and LSND can be solved if we consider neutrinos decay, as, in this model, we have heavy neutrinos, if we introduce the scalar singlet ϕ_2 the model still mimic a 3+2 model and it have a heavy neutrino also, see the analysis of the VEV's below the eq. (1), thus the neutrinos decay can be considered to explain the problem of appearance and disappearance [12].

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